Heather T. Gould

# Building understanding of fractions with LEGO® bricks

**Why use these** well-loved manipulatives to teach students to understand fractions? Using manipulatives to demonstrate mathematical concepts can be an extraordinarily effective teaching technique to develop students' understanding. Experienced teachers recognize that for manipulatives to work effectively, activities must engage students and activate their prior knowledge and intuition. Using LEGO bricks in the mathematics classroom not only fulfills these requirements for a good lesson with manipulatives, they can also teach a wide variety of mathematical concepts at a relatively low cost.

Teaching mathematics with LEGO bricks offers several benefits. First, children often become more receptive to an activity if they feel they are connected to it by some nonacademic means. They know and love playing with LEGO bricks. Second, because children love the bricks and often play with them at home, a teacher can incorporate the children's intuition regarding the use of the toys. With little effort on the teacher's part, this intuition can be developed into mathematical knowledge. Third, with increasingly limited school budgets, many teachers find it necessary to purchase their own educational materials. Maintaining a relatively small classroom collection of LEGO bricks and using them for a wide variety of mathematical activities can drastically reduce material costs and eliminate the need for single-use items.

In this article, I focus on using LEGO bricks to help students learn about fractions, with an emphasis on equivalent fractions. Using the bricks, one can teach concepts such as fractions as parts of a whole, equivalent fractions, and arithmetic with fractions. The lesson discussed here was implemented in fourth grade and college algebra classes, and everyone, regardless of age, appreciated the integration of these toys into the mathematics classroom.

This student sorted her set of bricks by their size.

HEATHER T. GOULD

## Introducing fractions

Before reading a discussion of how to teach about equivalent fractions with LEGO bricks, consider some terminology and how students can learn to represent fractions with the bricks.

### Terms

A *stud* of a LEGO brick refers to the round peg on top, which helps connect bricks. We name a brick by the number and orientation of the studs: The number of studs across the length of the brick ($l$) and the width of the brick ($w$)

FIGURE 1

Twelve 1 × 1 bricks cover the whole.

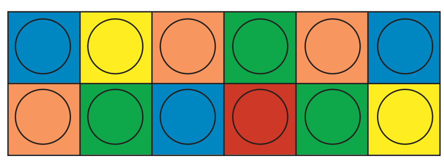
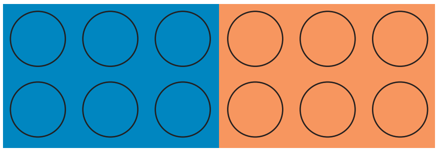


FIGURE 2

Two 2 × 3 bricks cover the whole.



name the brick as $l \times w$. For example, a brick with six total studs could be called a *one-by-six brick* or a *two-by-three brick*, depending on the arrangement of the studs.

## Naming fractions

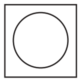
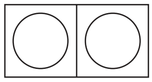
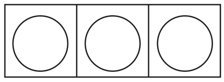
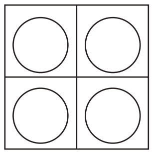
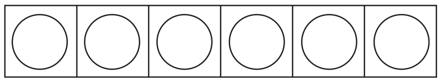
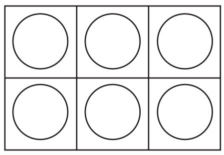
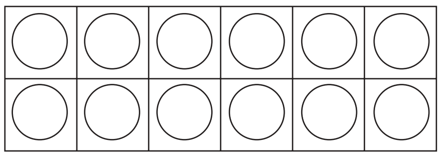
To begin the lesson, students looked through their set of bricks and made note of the types of bricks they possessed. Each group of students had one 2 × 6 brick, two 1 × 6 bricks, two 2 × 3 bricks, three 2 × 2 bricks, four 1 × 3 bricks, six 1 × 2 bricks, and twelve 1 × 1 bricks. This set is enough for students to learn about wholes, halves, thirds, quarters, sixths, and twelfths. I instructed students to find the 2 × 6 brick and explained that this brick would be called the *whole* for now. Then I asked the groups to use the 1 × 1 bricks to completely cover the whole brick. Clearly, there were twelve 1 × 1 bricks covering the whole (see **fig. 1**). First, in small groups, and then as an entire class, I had students discuss what might be a good fraction name for these 1 × 1 bricks. Students provided such answers as *one-by-one, one*, and *small square*. By reminding students that fractions help us relate the size (or part or proportion) of one thing to another, we reached the desired result of labeling these bricks as *one-twelfth* (1/12).

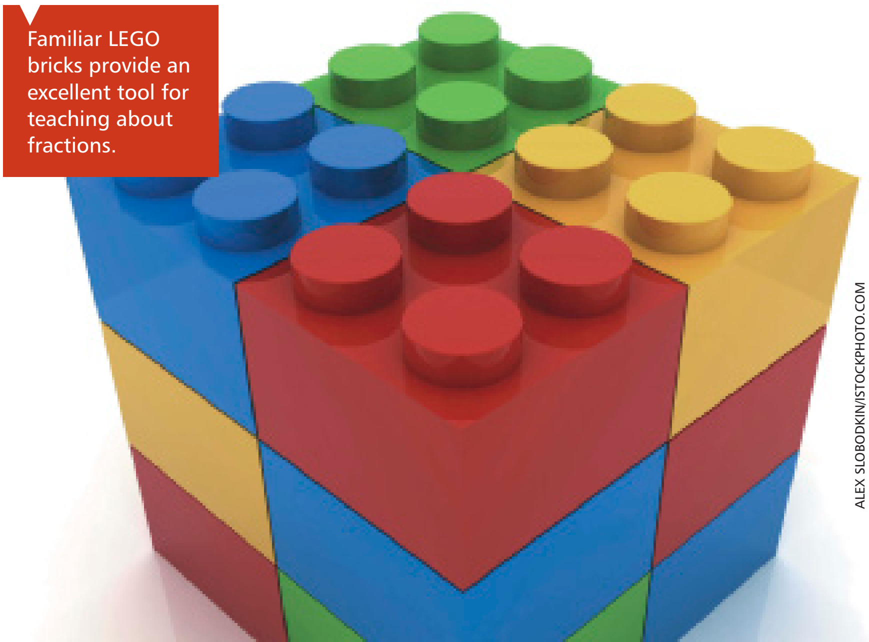
I repeated the process with the 2 × 3 brick, whose fraction name is the *one-half* (1/2) *brick* (see **fig. 2**). It was important to repeat this process with different bricks as many times as necessary to ensure that students understood the reason these bricks were named as they were in the "fraction world." See **table 1** for a complete list of the bricks' names in relation to the 2 × 6 whole brick.

Students occasionally confused the number of studs on the brick for the name of the fraction. For example, a 2 × 2 brick representing 1/3 has four studs. Students wanted to call this brick *one-fourth* (1/4) because of the number

TABLE 1

To record their findings, students in fourth grade and college algebra classes used a table similar to the one below.

| Picture of brick | Brick name | Fraction name |
|---|---|---|
|  | 1 × 1 | $\frac{1}{12}$ |
|  | 1 × 2 | $\frac{1}{6}$ |
|  | 1 × 3 | $\frac{1}{4}$ |
|  | 2 × 2 | $\frac{1}{3}$ |
|  | 1 × 6 | $\frac{1}{2}$ |
|  | 2 × 3 | $\frac{1}{2}$ |
|  | 2 × 6 | 1, or Whole |



Familiar LEGO bricks provide an excellent tool for teaching about fractions.

ALEX SLOBODKIN/ISTOCKPHOTO.COM

of studs they saw. Emphasizing the idea that exactly three of these types of bricks will cover the whole, and that is what makes this brick one part of three, helped correct this common misconception. I found it useful to end the lesson at this point and return to it the next day, giving students some time to think about the fraction-naming schemes.

## Equivalent fractions

Pretending that a LEGO king needed to build his castle, I instructed each group to give up two 1 × 1 bricks as a tax payment. With the remaining bricks, students were to cover as much of the whole with 1/12 bricks as possible and then find one brick that covered the missing spot perfectly. Letting them know that rearrangement of the 1/12 bricks was allowed was important. The underlying question to be answered here was, Which fraction brick can be used to “stand in” for two 1/12 bricks, otherwise known as 2/12? The desired conclusion is that one 1 × 2 brick, or a 1/6 brick, would work in this case (see **fig. 3**). By emphasizing that two 1/12 bricks represent the fraction 2/12, I led another discussion with the following prompts:

- **What** makes the 1/6 brick and two 1/12 bricks compatible so that they cover the same area?
- **Why** do these work together so nicely?
- **What** could be said about their numerators and denominators?
- **Do** you see anything interesting about these fractions?

The fourth graders found surprising answers to my questions. For example, most had noted that 1 + 1 = 2, and 6 + 6 = 12, and through class discussion, students realized that doubling the numerator and denominator of 1/6 results in the representation 2/12. A clever fourth grader used cross multiplication on the number sentence 1/6 = 2/12 to find equivalence between the products of the means and the extremes. She mentioned this because she saw it as an interesting occurrence, not because she recognized an important mathematical identity. As the examples show, the answers to the discussion questions laid the foundation for a beginning understanding of equivalent fractions.

FIGURE 3

The 1 × 2 brick on the right represents $\frac{2}{12} = \frac{1}{6}$.

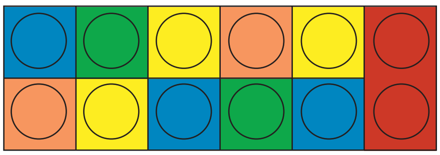


FIGURE 4

The 1 × 3 brick on the lower right represents $\frac{3}{12} = \frac{1}{4}$.

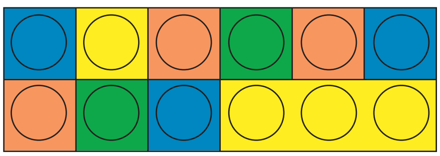
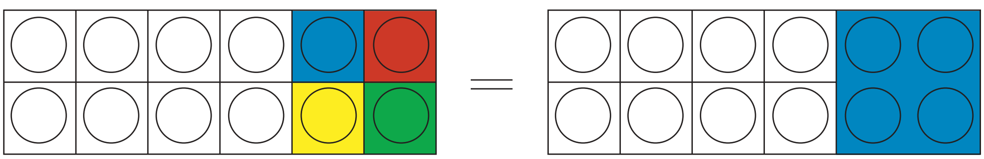


FIGURE 5

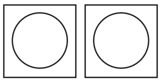
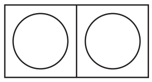
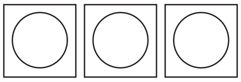
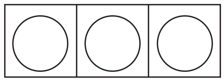
The findings for the LEGO bricks shown could be recorded as the fractions beneath them.



$$\frac{4}{12} = \frac{1}{3}$$

**TABLE 2**

Students recorded their equivalent fractions to complete a chart.

| Missing bricks | Mathematical name of these missing bricks | Drawing of single brick that replaces missing bricks | Replacement brick's mathematical name | Equivalent fraction equation (col. 2 = col. 4) |
|---|---|---|---|---|
|  | $\frac{2}{12}$ |  | $\frac{1}{6}$ | $\frac{2}{12} = \frac{1}{6}$ |
|  | $\frac{3}{12}$ |  | $\frac{1}{4}$ | $\frac{3}{12} = \frac{1}{4}$ |

Upon completion of this round of discussion, the groups removed the 1 × 2 brick from their whole brick. Then half the groups removed one additional 1 × 1 brick, while the other groups removed two more 1 × 1 bricks so that some students were working with quarters and some were working with thirds. In both cases, I asked students to find just one single brick that covered the missing space. (See **fig. 4** for a depiction of the first case, where students were missing three 1 × 1 bricks.)

After the groups determined the correct brick to cover the whole brick completely, they prepared a short presentation for the class about how they chose the correct brick to use, what that brick means in terms of fractions, and how their choice of brick relates to the fraction representing the number of 1 × 1 bricks they had lost. For example, a group that was missing four 1 × 1 bricks showed the class that four 1 × 1 bricks would fit onto a 2 × 2 brick perfectly (see **fig. 5**). They explained that because the 1 × 1 bricks

One whole brick can be covered with twelve 1 × 1 bricks.

The white 2 × 2 brick represents one-third of the whole brick.

HEATHER T. GOULD

would cover the 2 × 2 brick perfectly, then the 2 × 2 brick would work to cover the space that was missing four 1 × 1 bricks. Thus, they concluded that 4/12 = 1/3 (see **table 2**).

I continued the lesson by removing different numbers of 1 × 1 bricks. For example, removing six 1 × 1 bricks in total gave students the chance to find the equivalence of 6/12 and 1/2. Here, students found that two bricks, namely, the 1 × 6 and the 2 × 3, would work. When asked why two different bricks would work to represent the same fraction, the students explained to me that both bricks were made up of six studs, so they had the same fractional meaning.

### Differentiating

I took away eight 1 × 1 bricks to challenge some students who seemed to understand the concept well. They found that a single brick would not work to cover the space. They tried combining different types of bricks. I asked them to try using all the same type of brick to cover the space, even if they needed more than one such brick. They found that they needed to use two 2 × 2 bricks or four 1 × 2 bricks to make 2/3 (see **fig. 6** and **fig. 7**). This exercise allowed them to experience three separate fraction names, representing the same part of the whole, as well as equivalent fractions where one was not a unit fraction. This step is particularly useful to differentiate learning for students who are ready to discover how to find common factors in the numerator and denominator so they can recognize equivalent fractions from written representations. One teacher reported that seeing the "lurking common factors" in several representations of the same fraction helped students see factors as causing the equivalence rather than being coincidental in the representations.

### Changing the whole

Once students understood the concept of equivalent fractions related to twelfths, I changed the size of the whole brick. By seeing the size of the *whole* change, students come to understand that each brick can represent several different fractions, depending on the size of the whole brick to which it is being compared—similar to understanding that one foot represents both 1/3 of a yard and also 1/5280 of a mile. I led a discussion on what the appropriate fraction names are for each of the bricks with this new whole. For example, if a 2 × 4 brick is used as a whole, a 1 × 2 brick that represents 1/6 of the earlier whole will now represent 1/4. This concept challenged some students because once the size of the whole changed, the fractional names of bricks also changed. This is a good misstep for a student to make because clearing up this confusion reinforces the idea that a fraction represents a part of a whole.

FIGURE 6

Two 2 × 2 bricks represent $\frac{8}{12} = \frac{2}{3}$.

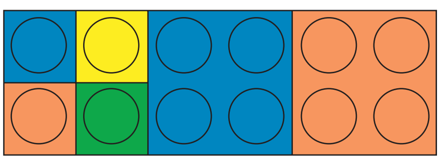
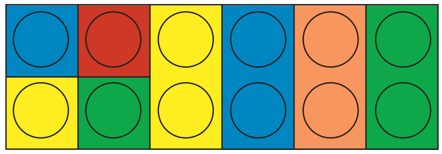


FIGURE 7

Four 1 × 2 bricks represent $\frac{8}{12} = \frac{4}{6}$.



Once this misunderstanding was corrected and we had changed the size of the whole brick, I asked students to first guess which fractions are equivalent to which. Then I had them give short explanations before physically manipulating the bricks. In the discussions that followed, I had groups explain their guesses, their justifications, their results, and why their guesses were right or wrong. The biggest misconception students had was in still wanting to keep the same fraction name for the same brick, regardless of the size of the whole. In general, though, students were able to use their prior knowledge of how to fill in an empty space with LEGO bricks to come to the correct conclusions; they were able to see that, for example, two 1 × 2 bricks would cover the same amount of space as one 2 × 2 and that these two sets must represent equivalent fractions.

## Moving ahead

To build on their understanding of fractions using the LEGO bricks, I engaged students in performing fractional arithmetic, such as addition and subtraction, using the bricks. I asked students to add by finding the correct number of studs that each fraction being summed represents and use the corresponding number of $1 \times 1$ bricks to combine them. Using this method with the $2 \times 6$ brick as a whole for the sum of 1/4 and 1/6, students placed five $1 \times 1$ bricks—three to represent 1/4, and two to represent 1/6—and concluded that $1/4 + 1/6 = 3/12 + 2/12 = 5/12$. Subtraction works in a similar manner.

Simplifying fractions after arithmetic has been performed is as easy as "economizing" bricks. This means that once students have found the correct number of $1 \times 1$ bricks in the sum or difference, they try to find one type of brick capable of representing those $1 \times 1$ bricks. But since the bricks they find are larger than the $1 \times 1$ bricks, they will be using fewer bricks in total. This "economizes" the bricks and achieves simplification. For example, again using the $2 \times 6$ brick as a whole to subtract 1/3 from 1/2, students could create the equivalent of 1/2 with six $1 \times 1$ bricks, then remove four $1 \times 1$ bricks to represent subtracting 1/3. They would then look for one fraction brick (or one type of fraction brick) that covers this same amount of space, finding that a $1 \times 2$ brick representing 1/6 will work. So they have used the fewest number of bricks possible, and the bricks are economized.

Regardless of *how* you choose to continue using your classroom collection of bricks, continuing to build on students' prior knowledge is important. Children know how to manipulate the bricks well before entering the classroom. Thus the most important learning activity within these LEGO lessons is supporting the youngsters' understanding and knowledge of fractions.

## Conclusion

See how easy it is to adapt one of your students' favorite toys into a valuable mathematics lesson? The concepts derived from this lesson build on students' prior knowledge and intuition. And, because students enjoy playing with the bricks, they happily allow the knowledge to build.



The red $1 \times 3$ brick represents one-fourth of the whole brick.

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