Bowland maths : Alien invasion

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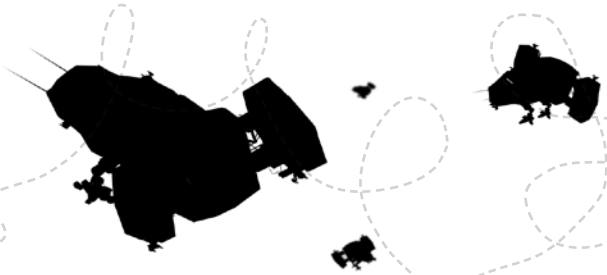

Previous learning

Before they start, pupils should be able to:

- use and interpret coordinates in all four quadrants (level 5);
- recognise simple properties of parallelograms and circles (level 5);
- calculate with decimals, using a calculator where appropriate (levels 4/5);
- solve simple problems involving ratio (level 5);
- identify multiples, square numbers, triangular numbers, powers of 2 (level 5);
- construct, express in symbolic form, and use simple formulae (level 5);
- construct and interpret data in simple line graphs (level 4);
- understand and use the mode and range to describe sets of data (level 4).

Main objectives at NC levels 5 and 6

In this unit, pupils learn to:

- appreciate mathematics as an enjoyable activity;
- use and apply mathematics to solve problems in familiar and unfamiliar contexts;
- appreciate some applications of mathematics;
- recognise that mathematics can be represented in different ways;
- work logically towards results and solutions;
- calculate accurately, using a calculator where appropriate;
- make connections within mathematics;
- estimate, approximate and check working;
- engage in mathematical discussion of results;
- relate findings to the original context;

and to:

Lesson 1
- estimate and calculate using measures in everyday situations (level 5);
- use and interpret maps and scale drawings (levels 5 and 6);
- recognise that all points on the circumference of a circle are the same distance from the centre (level 6);

Lesson 2
- use the mean, median or mode (level 5);
- use all four operations with decimals to two places (level 5);
- solve problems involving direct proportion (levels 5 and 6);

Lesson 3
- discuss and interpret graphs arising from real situations (levels 5 and 6)

Lesson 4
- generate integer sequences (levels 4 and 5);
- find and describe the *n*th term of a sequence (level 6);
- formulate and solve linear equations (level 6).

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Lessons

1 The landing
2 The plan
3 Alien behaviour
4 The escape

About this unit

This unit consists of four lessons of 50 minutes to 1 hour.

The lessons are based on a full-scale alien invasion coinciding with a school visit to a city. The invasion leads to a series of non-routine problems for pupils to solve by working in small groups. The problems are linked to the theme of mathematical communication and are designed to promote discussion, reasoning and creativity. The mathematical topics involved are not always immediately obvious as they are set in an unusual context. However, they are all part of the KS3 mathematics curriculum, mainly at National Curriculum levels 5 and 6. Each lesson ends with a 'cliff hanger' intended to maintain pupils' interest and to lead into homework tasks.

The unit includes these introductory notes, four sets of lesson notes accompanied by animated video and audio clips, slides, resource sheets which include homework tasks, and solutions to the problems. The first homework is essential to the progress of the unit but the rest are optional.

Presenting the unit to pupils

The mathematics in the unit is suitable for pupils working at National Curriculum levels 5 and 6, i.e. pupils of average and above average attainment in Years 8 or 9, and high attaining Year 7 pupils.

The unit can be an opportunity for pupils to apply and use skills that they have previously been taught and to see connections between mathematical topics. In this case, the lessons are best taught consecutively.

Alternatively, the unit can be an opportunity to introduce or extend skills and be taught, say, as one lesson per week for four or more weeks. In this case, the intervening lessons can be used for further teaching and practice.

In classroom trials of Alien invasion, the most engaging and dynamic lessons were those where teachers presented the unit as though it were they and their classes who were making the school visit, using words such as 'you' and 'we' instead of 'they'. The problems became impelling and the pupils were highly motivated to solve them. They felt part of the situation rather than removed from the action.

The pupils' favourite parts of the lessons were the videos, closely followed by the map work and the codes. Here are some of their reactions:

'We worked as a team and it was fun.' (Year 7, set 1)
'The lessons were exciting.' (Year 7, set 1)
'The problems stretch your mind.' (Year 8, set 1)
'The maths wasn't forced – we didn't know what we were learning until we thought about it.' (Year 8, set 1)
'We covered all areas of maths in a fun way.' (Year 8, set 2)
'I would like more lessons like this because they were interactive and a bit of a challenge. It's a better way to learn.' (Year 9, set 2)
'I like doing something different to normal boring lessons.' (Year 9, set 2)
'It's much better than working out of books.' (Year 9, set 3)

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Adapting the lessons

Each set of lesson notes starts by explaining to all teachers how the video, audio and print resources relate to the storyline, and the possible learning points for the lesson. These notes are on a shaded background.

The rest of the lesson notes are a guide to less experienced teachers or teachers from non-conventional backgrounds on the possible flow of the lessons, questions to ask and adaptations of activities for pupils of differing abilities.

However, all teachers should feel free to tailor the lessons, activities and print resources to the particular needs of their classes. For example:

Optional activities

Since lessons vary in duration from one school to another, in all four lessons some activities and resources are optional. These can be omitted if lessons are 50 minutes or less and teachers so wish.

The optional parts of lessons are shown in italics in the lesson notes.

Supplementary problems

The supplementary problems are additional materials for pupils who finish the main tasks quickly. As such, these too are optional.

Differentiated activities

Ideas for simplifying or extending the main problems are given in each lesson plan, including suggestions for the type of support that might be needed.

Homework

The homework tasks, which are designed to reinforce the learning in the lesson, are labelled Task A, Task B and, for lesson 1, Task C. This provides flexibility through options for shorter or longer homework, depending on the school's policy.

All the homework tasks are optional, with the exception of Task A of lesson 1, which involves pupils in gathering data to work out their personal average walking speed. This data is used in lesson 2.

If the supplementary problems have not been tackled in the lessons, these could be offered as an alternative to the homework tasks.

Modifying lesson 1

In trials, a teacher whose pupils were unaccustomed to problem solving decided to adapt the first lesson and homework to form lesson 1a and lesson 1b. This allowed more time for pupils to solve the problems and for teaching of the associated skills. It also ensured that the essential homework was completed by all pupils. This is her description of how she took the final part and homework of lesson 1 to make it into an extra lesson.

> *'In the extra lesson, I played the video clip (Resource 1.8) and gave out copies of the homework sheet (Resource 1.9). We simulated walking through busy streets – we timed a minute twice and counted the number of paces. This enabled pupils to answer the homework questions, which are good questions at an appropriate level.*
>
> *Then everyone entered their speed in mph and km/h into a spreadsheet I had prepared. I showed them how to use the spreadsheet to find the mean, mode and median walking speeds for the class and we discussed which would be best to use, and why. Pupils then worked in their groups with a laptop, and repeated this for their group.*
>
> *I ended the extra lesson by discussing with the class the factors that could affect the average walking speed of an individual person, and then factors that could affect the overall performance for a group of people walking in the same conditions.'*

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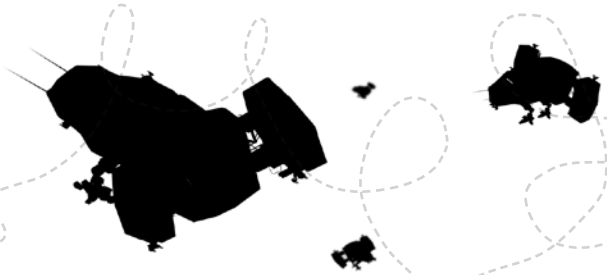

Modifying lesson 4

A few teachers in the trials found that for pupils who are currently working at level 5, the fourth lesson was better if taught over two periods. This allowed them to build in some preliminary teaching or revision of work on deriving formulae and finding the *n*th term of a sequence.

One trial school invited pupils from Years 10 to 12 to support less secure pupils in the fourth lesson. This proved to be a very successful way of making sure that all pupils completed the task and felt a sense of achievement. If you choose to do this, you may need to spend half an hour preparing the support students in how to help and, in particular, the questioning strategies to use.

Cross-curricular opportunities

Lesson	Subject	Topic
1	Geography	Map work; environmental study, e.g. 'Why Aliens chose our planet'
	English	Creative writing about the alien landing
2	ICT	Using LOGO to generate and transform shapes
	PE	Gymnastics; navigating an obstacle course; calculating average walking and running speeds with spreadsheets
	English	Descriptions of the scene
3	Science	Animal behaviour; cooling curves
	CDT	Nutrition analyses
4	Art	Study of patterns
	English	Report writing, e.g. for a radio news broadcast
	ICT	Taking and editing digital photographs, e.g. using Photoshop; using email to send report

Practical resources

for the teacher, whiteboard, data projector and laptop

spreadsheet software (optional)

for pupils, string, ruler, compasses, scissors, calculators, small counters

Key terms and notation

problem, solution, method, pattern, relationship, rule, represent, solve, explain, justify, reasons, reasoning

estimate, approximately, round, decimal place, accuracy, calculate, multiple, product, quotient, scale, scale factor, direct proportion, ratio

units of measurement, metric, imperial, compass directions

graph, linear, straight-line graph, distance-time graph, formula, equation, expression, sequence, term, position-to-term rule, *n*th term

mean, median, mode, data

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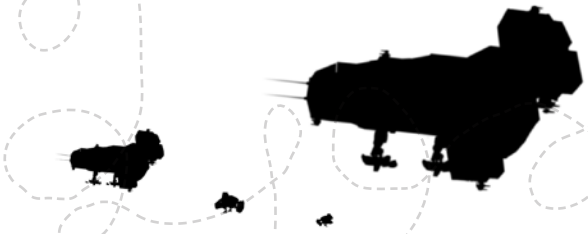

Common errors and misconceptions

- Pupils have no strategies for estimating measurements, such as using benchmarks.
- Pupils choose inappropriate units for making a measurement.
- Pupils don't recognise equivalent ratios, or make errors when they scale quantities up or down.
- Pupils think that increasing a map scale increases the map distance rather than decreasing the map distance.
- Pupils misinterpret the scale on an axis of a graph, or think that a positive gradient on a graph represents 'going up hill'.
- Pupils don't distinguish between sketching a graph and drawing it accurately.
- Pupils confuse term-to-term rules and position-to-term rules for sequences.

Assessment

You could present the objectives for the unit worded in pupil-friendly language for pupils to make a self-assessment of the progress that they think they have made.

For the final homework, some suggestions are made for relevant National Curriculum test questions. You could ask pupils to try a selection of these questions to help you and them to judge how effective their learning has been.

Useful websites

Secondary Strategy resources, including Framework:
www.standards.dfes.gov.uk/keystage3/

UFOs
news.bbc.co.uk/1/hi/uk/4981720.stm
en.wikipedia.org/wiki/UFOs
homepage.ntlworld.com/mjpowell/ufo.htm

Average walking speeds
www.go4awalk.com/navigationskills/timing.php
www.routehiker.org.uk/statistics.php
www.comp.leeds.ac.uk/kwb/ENV/conf_draft/sc06lcn.pdf

Road traffic speeds
www.dft.gov.uk/pgr/statistics/datatablespublications/roadstraffic/speedscongestion/

Pascal's and Sierpinski's triangles
mathforum.org/dr.math/faq/faq.pascal.triangle.html
math.rice.edu/~lanius/fractals/pas2.html

Fibonacci
www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html

Codes and cryptography
en.wikipedia.org/wiki/Codes

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Narrative

While the class is visiting the city of Manford, monstrous alien spaceships appear in the sky and drift overhead before slowly landing. The class has been separated into groups to visit various places of interest, so the first aim is to try and join up and escape on the school bus. However, the landing of the ships has caused a lot of panic. Everyone is trying to leave the city, clogging the streets with traffic.

Problem

Where exactly have the Alien ships landed?

Mathematics content objectives

- Estimate and calculate using measures in everyday situations (level 5)
- Use and interpret maps and scale drawings (levels 5 and 6)
- Recognise that all points on the circumference of a circle are the same distance from the centre (level 6)

Learning points

- A map ratio is the ratio of the distance on the map to the actual distance on the ground, in the form of a unitary ratio 1 : *n*, without units, e.g. a scale of 1 cm to 50 m is a map ratio of 1 : 5000, since 50 m is 5000 cm.
- The scale of a map or drawing should always be stated next to the map.
- A distance 'as the crow flies' is the shortest distance between two points, measured as a straight line.
- Points on the circumference of a circle are an equal distance from the centre.
- 8 kilometres is approximately 5 miles.

Alien invasion resources

1.1 A4 resource sheet of tourist map marked in squares
(print one per pair, preferably in colour, but black and white is acceptable as the colour version is shown on the screen)

1.2 Video clip: Globe breaking news – presenter announces the arrival of four large spaceships over Manford City (3 minutes)

1.3 Slide: tourist map of the city marked in squares

1.4 Optional video clip: amateur video of alien ships overhead, cars blocking streets (2 minutes)

1.5 An A4 resource sheet with gaps in the clues to be filled in
(print one per pupil)

1.6 Audio clip: radio newsflashes eyewitness accounts giving clues which allow the positions of the alien ships to be mapped (4 minutes)

1.7 Slide: table of equivalent distances in miles and kilometres with the distances as represented on the map

1.8 Optional A4 resource sheet with a supplementary problem for groups that finish quickly (print several copies)

1.9 Video clip: doors opening on the spaceships (1 minute)

1.10 A4 resource sheet of homework task 1 (print one per pupil)

For pupils: rulers, string, calculators and small counters to mark positions on maps.

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Main activity

Give out **Resource 1.1**, a tourist map of Manford city, one per pair. Explain that you, a couple of other teachers and your class have arrived in Manford on a school bus. You have split up into groups to visit different places.

Ask the pairs to consider the map's features. Let the discussion get under way, then without explanation play **Resource 1.2**, a 3-minute video clip setting the scene. Four big space ships have loomed overhead and have landed out of sight.

► What are we going to do?

Display **Resource 1.3**, a slide of the tourist map. Elicit the need to get back to the bus and try to escape. *If time allows, play **Resource 1.4**, a 2-minute amateur video of spaceships overhead and cars blocking streets.*

► What information do we need before we make any move?

Establish that the first priority is to determine exactly where the ships have landed. You may need to point out that the map currently has no scale and north is to the right. Check that pupils can use the grid system by asking them for the positions of a few features.

Give out **Resource 1.5**, one per pupil. Explain that these are clues from the latest broadcast from the local radio station but that there are gaps in the information. Get pupils to study the gaps, then draw out from them that most of the missing information is likely to be distances or directions.

Play **Resource 1.6**, an audio clip of eye-witness accounts lasting 4 minutes. Ask the class to listen carefully and to fill in the gaps. Advise them to use abbreviations such as km for kilometres, m for miles, N for north, and on. You may need to play the audio clip a second time. (For the missing information, see the Solutions.)

Leave it to the groups to sort out the information and decide how to use it to locate the spaceships. You may need to remind them that 5 miles is about 8 km.

Differentiation

Some pupils may need help in identifying and using the map scale. If necessary, discuss and complete the table of information on **Resource 1.7**, a slide of distances as represented in centimetres on the map.

*If time allows, pupils could try the supplementary problem on **Resource 1.8**. Introduce this problem by saying that there could possibly be a side gate out of the zoo but, if so, more information about its whereabouts is needed.*

Review

Bring the whole class together to discuss methods and solutions. The scale of the map is 1 cm : 400 m, or 1 : 40 000, and the first three space ships are at G8, L10 and O6. Discuss the three possibilities for the position of the fourth space ship at the vertex of a parallelogram, at D12, T8 and J4. D12 can be ruled out as the first ship, at G8, was the only one to land anywhere south of the Observatory. T8 can be ruled out as it is due north of St Andrew's Cathedral, contradicting one of the clues.

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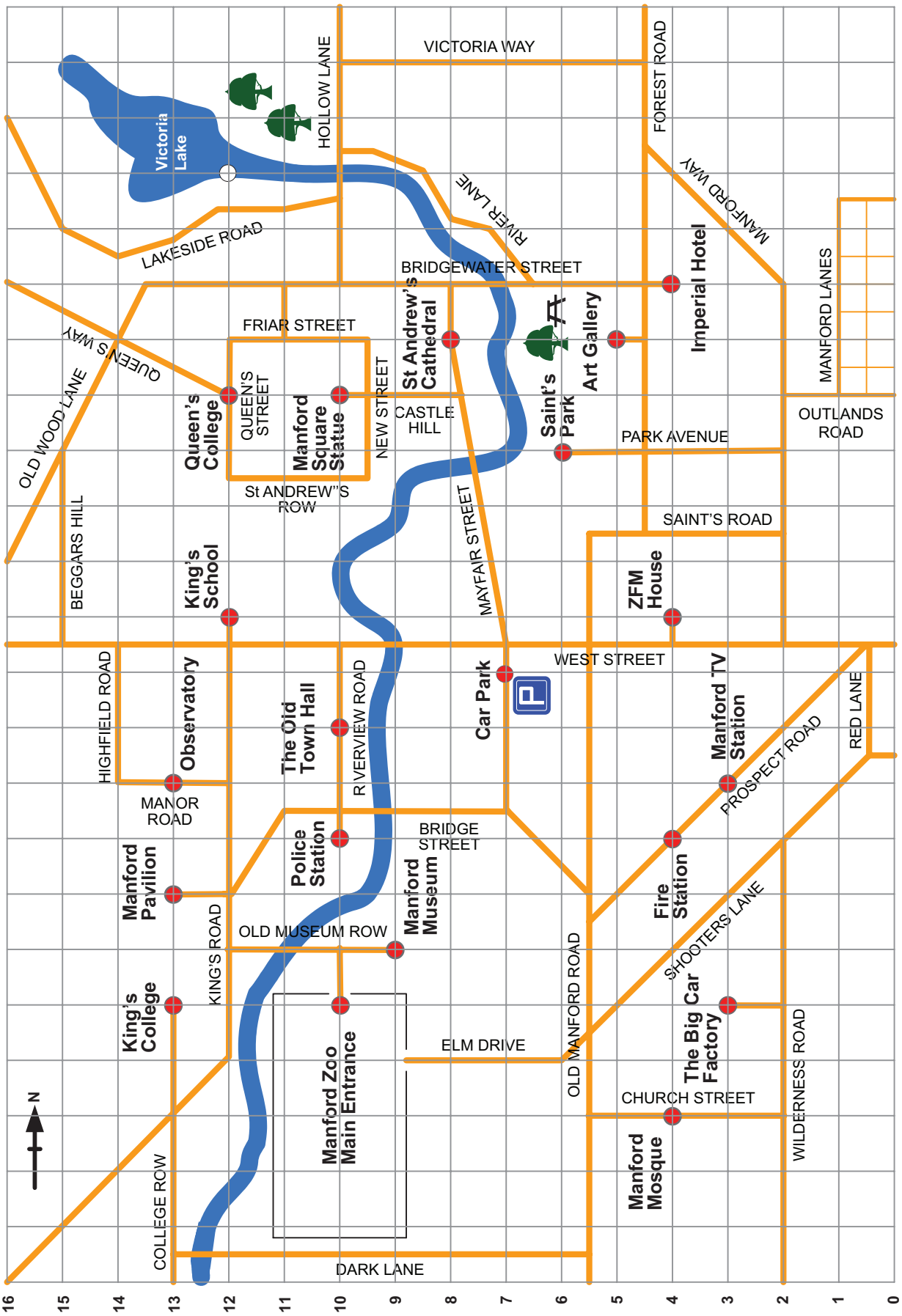
Homework

Play **Resource 1.9**, a 1-minute video clip. The large metallic doors on the alien craft open and in the darkness beyond them strange noises can be heard.

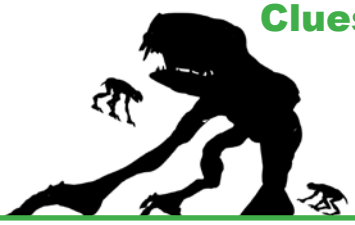
Give out copies of **Resource 1.10**. Task A is essential, as pupils collect data that they will need in the next lesson. As an alternative, cover the work in an extra lesson, as described in the introduction to the unit. To provide a longer or more challenging homework, choose from Task B or Task C.

Advise pupils which problems to answer on the sheet and which in their exercise books. Remind them that their answers in Task B are estimates and should be sensible (e.g. rounded to 1 decimal place). Task A is essential, as pupils collect data that they will need in the next lesson. As an alternative, cover the work in an extra lesson, as described in the introduction to the unit.

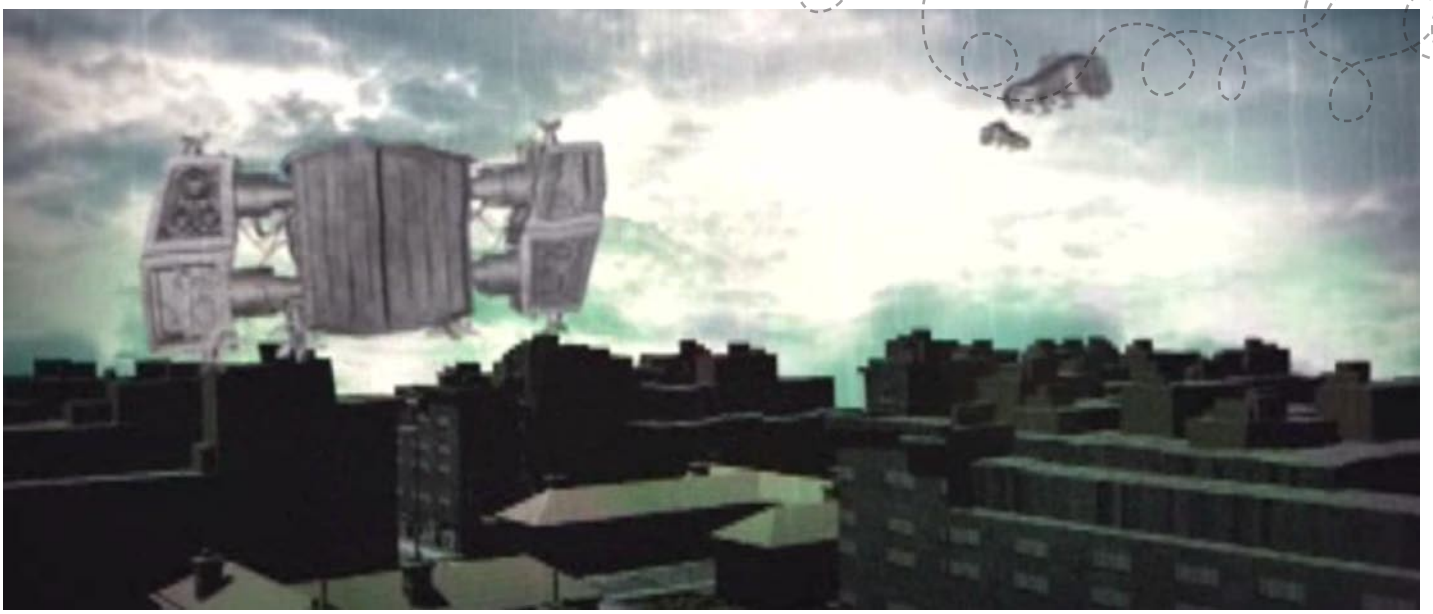
RESOURCE SHEET 1.1
Tourist map



RESOURCE SHEET 1.5
Clues



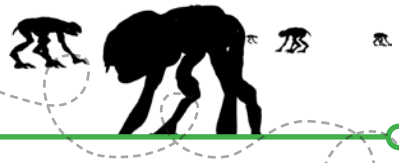
Clues



Listen to the radio broadcast and complete the clues.
Use them to work out the scale of the map and where the spaceships have landed.

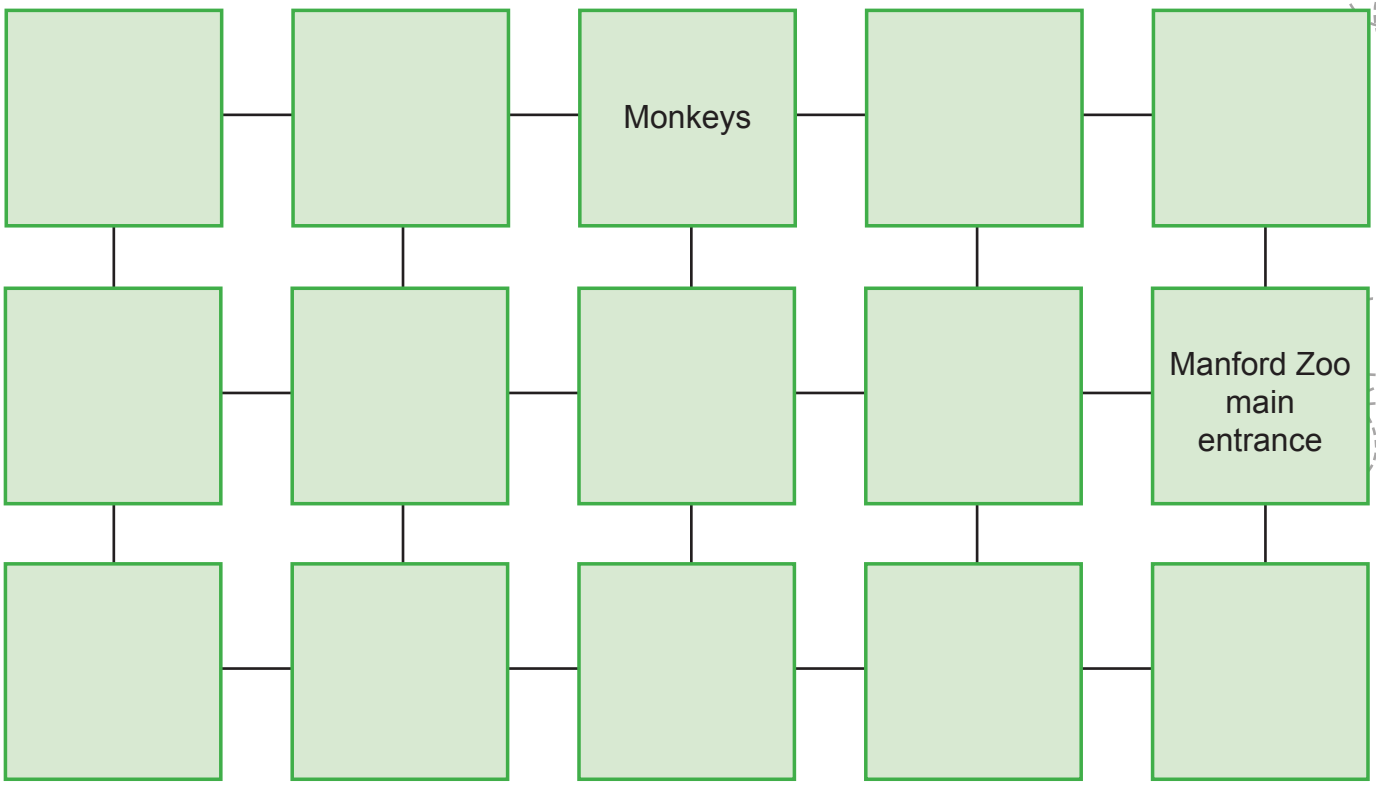
- ‘I’m standing on the roof of the Fire Station. I can see the statue in Manford Square _____ away as the crow flies on the other side of the river.’
- ‘We were both watching the first spaceship. Jen thought it landed _____ from the Mosque but Leela thought it landed _____ from Manford Museum.’
- ‘As far as I could tell, the second ship came down _____ north of the Police Station.’
- ‘I’m speaking to you from ZFM House. The third ship landed to the _____ of me at the same distance from King’s School as it is from Queen’s College.’
- ‘Whoever is behind these spaceships must have a plan. The ships have landed at the four corners of a _____’
- ‘I was watching from the Observatory as the spaceships landed. As far as I could judge, only the first spaceship landed anywhere to the _____ of the Observatory.’
- ‘From the top of the tower of St Andrew’s Cathedral, we couldn’t see any ships to the _____ of the tower but we watched one spaceship land due _____ of us.’
- ‘I think the _____ spaceship has come down very close to us at _____’

RESOURCE SHEET 1.8
Supplementary problem 1



Supplementary problem 1

The main entrance to Manford Zoo lies at F10 on the tourist map of the city. To see the monkeys, visitors must walk 800 metres south from the entrance and then 400 metres west.



The diagram shows the layout of the features of the Zoo.

Use the clues to work out where the various features are. Write them in the boxes.

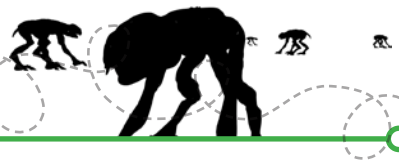
Clues

- The llamas are to the south-west of the main entrance.
- The main entrance is to the north-west of the goods entrance.
- The monkeys and the penguins are to the south-west of the polar bears.
- The zebras are to the south-east of the monkeys and to the north of the reptiles.
- The monkeys are to the north-west of the pandas and south of the camels.
- The aviary is next to the giraffes and south-west of the tea rooms.
- The dolphins are between the goods entrance and the side gate.

On the tourist map of Manford, what are the grid references for these?

a The side gate _____

b The giraffes _____



Homework 1

You will need a stopwatch or a clock or watch that has a second hand.

The Aliens have started to come out from the spaceships. To escape, your class is going to have to get back together as quickly as possible. To work out how long this will take, you will need to know your average walking speed.

Task A

There is a useful formula to estimate your average walking speed (s) in miles per hour:

$$s = \frac{n}{30}$$, where n is the number of steps you take in 1 minute

STEP 1	Time 1 minute. Count how many steps you take in 1 minute at the pace that you normally walk.	_____	
STEP 2	Divide the number of steps you take in 1 minute by 30.	_____	*For example, if you take 96 steps in 1 minute, you'll divide 96 by 30.*
STEP 3	Write down the result. Round it to 1 decimal place if necessary. This is the estimate of your average walking speed in **miles per hour**.	_____ mph	*For example, 96 ÷ 30 = 3.2, which means that you walk an average of 3.2 mph.*

It would be helpful to know what this is in **kilometres per hour**.
Remember that 5 miles is about the same as 8 kilometres, so 1 mile is 8 ÷ 5 = 1.6 kilometres.
So to change miles per hour to kilometres per hour, multiply the miles per hour by 1.6.

STEP 4	To estimate your average walking speed in kilometres per hour, multiply your speed in mph by 1.6. Round the result to 1 decimal place if necessary.	_____ km/h	*For example, a speed of 3.2 mph is roughly 3.2 × 1.6 = 5.12 km/h. This is 5.1 km/h to 1 d.p.*

Task B

1 How many **miles** do you think you could walk in 30 minutes?
2 How many **miles** do you think you could walk in 6 minutes?
3 How many **kilometres** do you think you could walk in 15 minutes?
4 How many **kilometres** do you think you could walk in 1 hour 20 minutes?

Task C

5 Write down three factors that could affect how fast you can walk.
If possible, look first at this website:
http://www.go4awalk.com/navigationskills/timing.php

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Narrative

The city is in chaos. Everyone is trying to leave and roads are congested. The green fog seems to be spreading. The Aliens are emerging from their ships and occupying large parts of the centre of Manford. It's essential for the class to meet up quickly in a safe place, avoiding the Aliens.

Problems

- Where should the muster point be?
- What's the shortest route to reach the muster point?
- If everyone walks at the average speed of his or her group how soon could everyone get there?

Mathematics content objectives

- Use the mean, median or mode (level 5)
- Use all four operations with decimals to two places (level 5)
- Solve problems involving direct proportion (levels 5 and 6)

Learning points

- Average speed is *distance travelled ÷ time taken*.
- The median is less affected by extreme values than the mean.
- The mode is only affected by how often values in the data set are repeated, not by what those values actually are.
- The mean gives an indication of all the values but it is more affected by extreme values than either the median or the mode.
- The mid-range (the mean of the smallest and largest values) can be useful as an estimate of the mean, especially when the data is uniformly distributed.

Alien invasion resources

2.1 Video clip: Alien expert interviewed on Globe news and advice for people in the City about avoiding the Aliens; shots of Aliens pouring out of their ships and people's descriptions of Aliens (4 minutes)
2.2 Slides: Maps and tables
2.3 A4 resource sheet of map of city streets (print one per pair)
2.4 Optional A4 resource sheet with a supplementary problem for groups that finish quickly (print several copies)
2.5 Audio clip: Mobile phone voicemail (45 seconds)
2.6 Video clip: Breaking news: 'The Aliens seem to be multiplying. Eye witnesses report the Aliens growing rapidly to twice their size before splitting into two. Press the red button now to vote whether you think this is the end of the world as we know it or text yes to 82255.' (1 minute)
2.7 Optional A4 resource sheet of homework task 2 (print one per pupil)

For pupils: string, ruler, compasses, pencils, calculators and small counters

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Main activity

Either before or at the start of the lesson, show or remind pupils how to estimate average walking speed using data from the homework. Discuss whether to use the mean, median or mode. The decision will probably depend on the size of the group and the distribution of the data. Discuss why an estimated average walking speed of 3.456 km/h is not sensible and why it should be rounded to 3.5 km/h.

Introduce the next stage of the invasion. Play **Resource 2.1**, a 4-minute video clip of a news announcement of Aliens leaving the ships, the green fog spreading, and interview with an alien expert.

Display **Resource 2.2**, **slide 1**, the street map of Manford, showing where the ships have landed. Give each pupil a copy of **Resource 2.3**, the map.

Say that the class has split up to visit different places of interest: King's College, Queen's College, the Art Gallery and the Big Car Factory. Tell each group which one of these places they are visiting, or choose other places if you prefer. Establish in discussion that as there is safety in numbers it would be best to meet up quickly somewhere central. Since it is getting dark, and rough ground will slow progress, it would be best to stick to the roads marked on the map. Pose these problems:

- Where exactly should the muster point be, and why?
- What is the shortest safe route to reach the muster point?
- What is the average walking speed of your group?
- If everyone in your group walks at your group's average walking speed, how long will it take for your group to get there?

Allow time for the groups to establish the circular 'no go' areas with radius half a mile/800 metres around each ship, to agree on their meeting point, and to work out the length of their shortest safe route by measuring and applying the map scale.

Show or remind pupils how to work out time, given a distance and the average speed by using the unitary method for direct proportion. In support, you could if you wish discuss and complete the tables on average walking speeds on **Resource 2.2**, **slides 2**, **3 and 4**.

The groups can now continue to solve the problems. Those that finish quickly could assist another group. *If time allows, they could try the supplementary problem on **Resource 2.4**.*

Differentiation

Allocate the shortest route to a central point such as the Car Park to pupils who would benefit from simpler calculations. They could also use the mid-range (the mean of the smallest and largest values) as an estimate for the average walking speed of their group, or be given an average walking speed to use such as 4 km/h.

For able pupils, mark points along their route with the height above sea level, to be taken into account. For example, it takes 1.2 minutes to walk 100 m at an average speed of 5 km/h. For each 10 m of ascent, add an extra minute, e.g. 300 m with 28 m of ascent will take 3 × 1.2 = 3.6 minutes, plus 2.8 minutes for going uphill.

Able pupils could also work out their time to cover each leg by rearranging the formula
speed = distance ÷ time to time = distance ÷ speed.

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Review

Display **Resource 2.2, slide 5**. Bring the whole class together to discuss and compare solutions and justify decisions. Establish that everyone has reached the muster point, and how long the first group had to wait until the last group arrived.

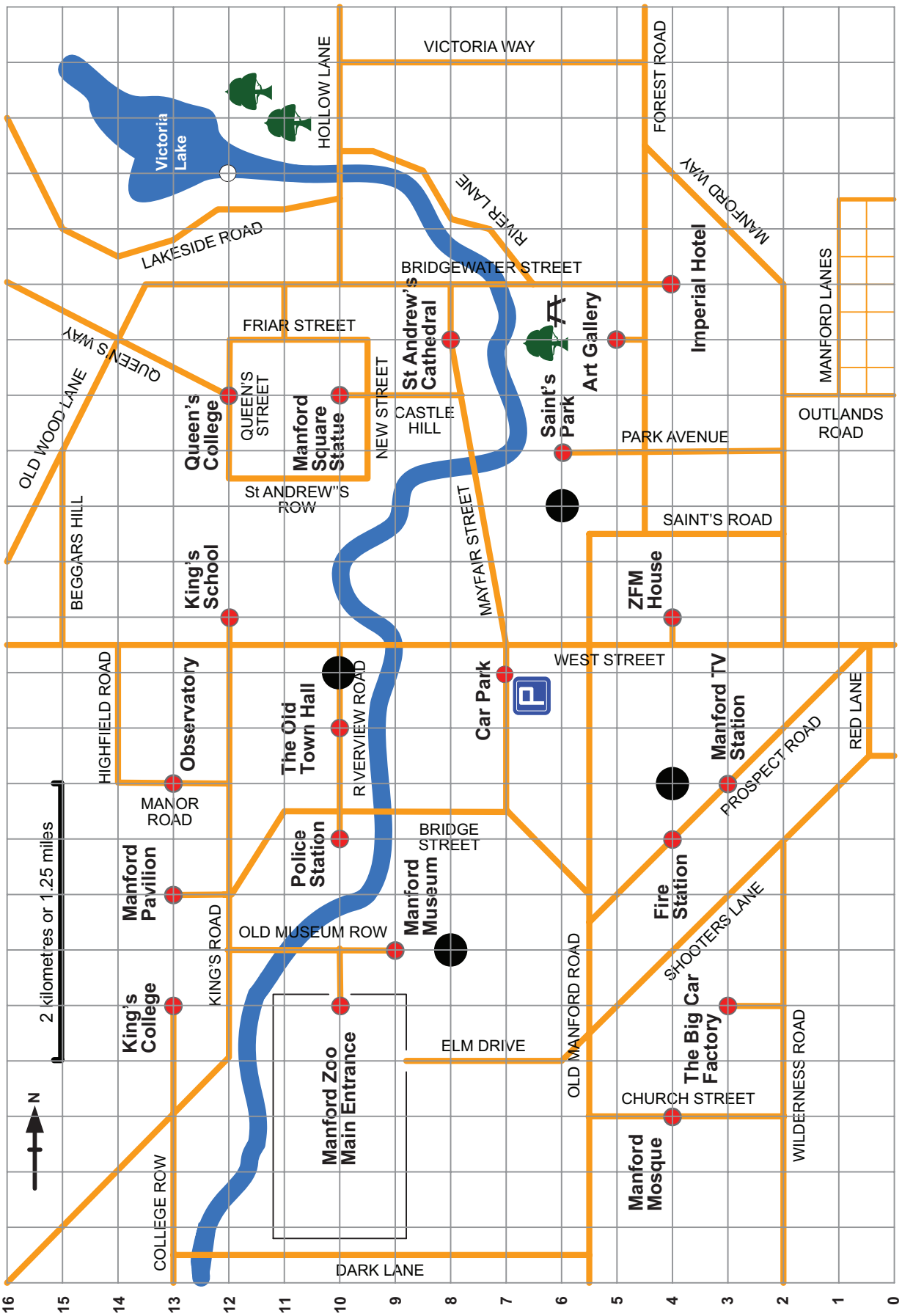
Homework

If the chosen muster point is not the Car Park, say that the whole class is now to go there. On the face of it, this may seem dangerous, but the bus is there and can be used for a speedy get-away before any more spaceships land.

Play **Resource 2.6**, a 1-minute video clip – the Aliens are multiplying!

Give out copies of **Resource 2.7**, choosing from homework Tasks A and B. For Task A, which is similar to the work done in class, pupils will need a copy of the street map. Task B is more challenging. The table can be completed on the sheet.

RESOURCE SHEET 2.3
Tourist map

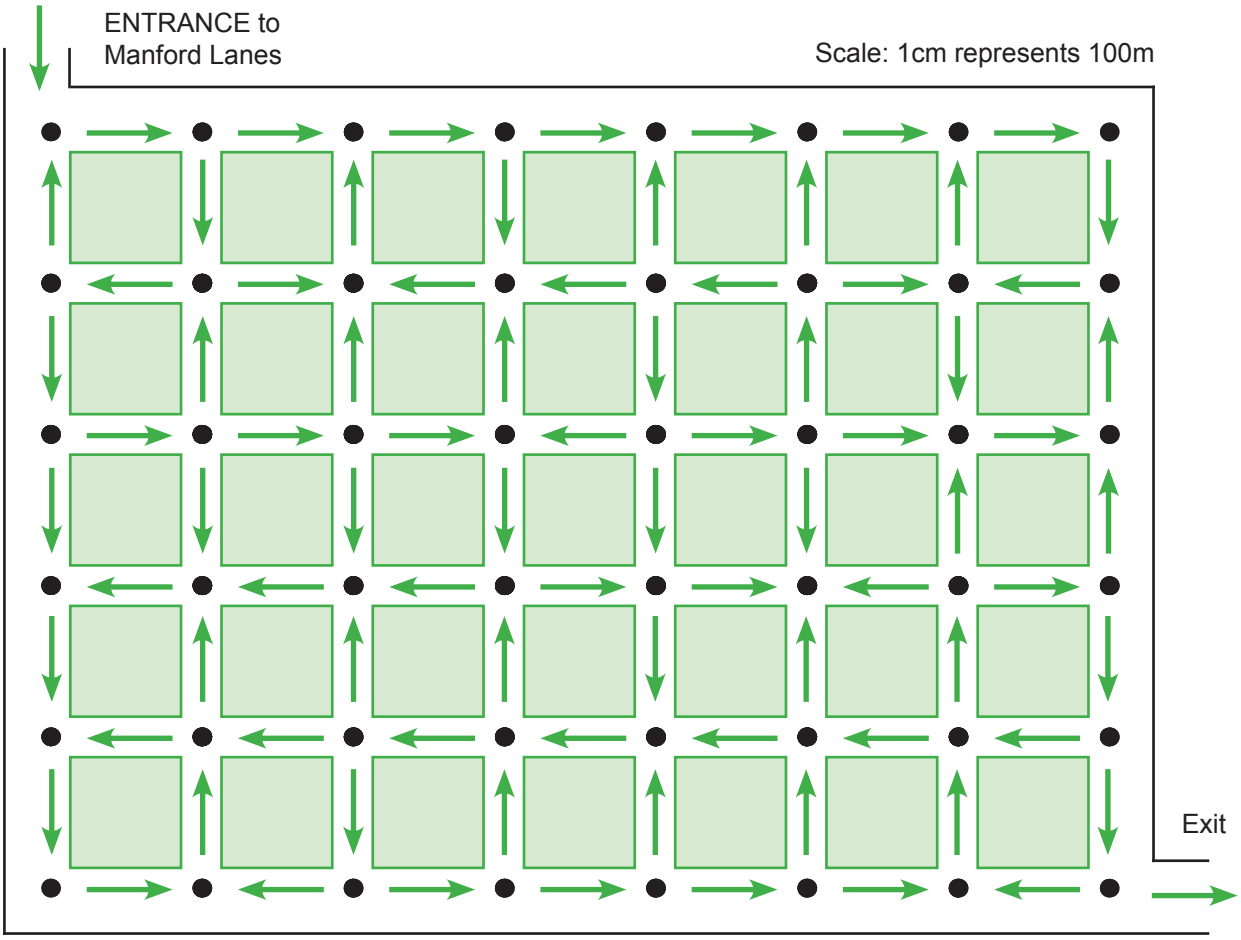


Supplementary problem 2

One group in your class has found its way to Manford Lanes.
This is an old area on the north-east side of the city.
Here the streets are too narrow for a spaceship to land.

The streets in Manford Lanes have got very crowded.
It is only possible to walk along them one way.

To escape from the city, you need to find a way to get from the ENTRANCE to the EXIT.



Each street junction is marked by a cone (the dots on the map).

a To escape from The Lanes, how far do you have to walk from the cone by the entrance to the cone by the exit? _____ kilometres

b How long will it take you to walk this distance at an average speed of 3 kilometres per hour? _____ minutes

RESOURCE SHEET 2.7
Homework 2

Task A

You need your street map of Manford.

Remember: 5 miles is about 8 kilometres

Everyone has made it to the car park where the school bus is waiting.

1 The whole class walking together can keep up an average speed of 2.5 miles an hour. How fast is this in kilometres per hour?

2 The school bus can travel in the congested streets at an average speed of 10 miles an hour. How fast is this in kilometres per hour?

3 Investigate the shortest safe distance by road from the Car Park to the Imperial Hotel.

4 How much quicker would it be to get to the Imperial Hotel by bus than by walking?

Task B

Two Aliens came out from each of the four spaceships at 10:00 am.

Every hour, each Alien has doubled in size and exploded into two.

Someone has been trying to predict how many Aliens there might be in a few hours time.

Complete this table for them.

Time	10:00	11:00	12:00	13:00	14:00	15:00	16:00
Number of Aliens	8 2^3	16 2^4	32				

a How many Aliens do you think that there will be 8 hours after they first came out of their spaceships?

b How many Aliens do you think there will be at midnight? Explain why.

c Write a formula to show the number n of Aliens that there will be h hours after they first came out of their spaceships.

BOWLAND MATHS

Narrative

The class has gathered at the top of the Imperial Hotel where they can see the whole city. The school has notified the Army that the class is stranded and a teacher has heard that everyone will be airlifted out during the night. The Army has asked for more information about the Aliens. The request is for information on the habits and movements of the Aliens to be sent to Army HQ.

Problems

- What information do we have about the behaviour of the Aliens?
- How can we represent the information so that it can be sent to HQ and interpreted quickly?

Mathematics Content Objective

- Discuss and interpret graphs arising from real situations (level 5)

Learning points

- A distance-time graph or travel graph describes stages in a journey.
- Before you interpret a distance-time graph, work out the scales on the axes.
- The horizontal axis is used for time.
- A sketch of a graph is not accurate but its shape gives a picture of what is happening to one variable in relation to the other.
- Two graphs of different stories or situations could have the same shape.

Alien invasion resources

3.1 Slides: Graphs for interpretation
3.2 Audio clip: Army commander requesting assistance (3 minutes)
3.3 Video clip: Globe news report with observations of the Aliens' behaviour and interviews with terrified citizens (4 minutes)
3.4 A4 resource sheets of a situation report to be copied and cut up to make packs of cards (make one pack per group of four pupils)
3.5 Optional A4 resource sheet with a supplementary problem for groups that finish quickly (print several copies)
3.6 Slides: Graphs to support discussion of situation report
3.7 Audio clip: Faint message from a teacher reporting her capture by the Aliens when she went outside to see if she could spot the helicopter (1 minute)
3.8 Optional A4 resource sheet of homework task 3 (print one per pupil)

For pupils: scissors to cut out the cards.

BOWLAND MATHS

Main activity

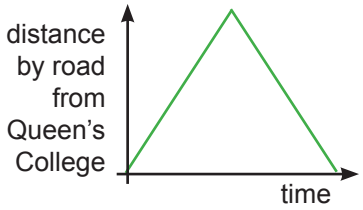
Either before or at the start of the lesson, show or remind pupils how to interpret a distance-time graph. Display **Resource 3.1**, **slide 1**, a graph representing the bus journey to Manford. Each section of the journey was at a constant speed (click on the slide to show each section in a different colour). Ask questions such as:

- How far is the school from Manford? How long did the journey take?
- How do we know from the graph when the bus was stationary? At what time did we stop? For how long?
- Which was the quickest part of the journey? How can you tell? What was our speed for this part of the journey?
- What kind of roads do you think we travelled on? (e.g. town and city roads for the first and last sections; a motorway for the middle two sections)

Invite someone to tell the story of the graph.

*If time allows, show the graphs on **Resource 3.1, slides 2 and 3**. Ask pupils to discuss in pairs what the story of each graph could be.*

Show **Resource 3.1**, **slide 4**, the map of Manford. Ask pupils to sketch a graph to represent the journey of someone walking at a constant speed around the square from Queen's College and back again. Make sure that pupils appreciate the difference between sketching a graph and drawing it accurately, and why the graph changes direction at the halfway point.



Introduce the next stage of the invasion. Explain that the class has travelled by bus to the Imperial Hotel and has gone to the top floor, which has access to the helipad on the roof. A teacher has heard that everyone will be airlifted out during the night and a message has been received from the Army Commander. Play **Resource 3.2**, a 3-minute audio clip of a request for information about Alien behaviour.

Discuss what you already know about the Aliens (e.g. they double in mass before exploding into two and need a green fog to support their existence). Say that a TV news broadcast is due, which may have more information. Play **Resource 3.3**, a 4-minute video clip reporting other bizarre behaviour and interviews with people who have had close encounters with the Aliens. Discuss the salient points.

Say that you have found what appears to be a report on the invasion, with graphs and information about the Aliens, but it has been cut up ready for shredding. If the groups could put it back together it could contain useful information for the Army.

Give each group a prepared set of cards made from **Resource 3.4**. *If time allows, groups could cut them out.* Ask the groups to match each item of information to its graph and to identify how to label the axes. When two groups have finished, ask them to join forces to compare and justify their decisions to each other.

Differentiation

For pupils who need more support, simplify the pack of cards by removing one or two of the harder matching pairs. When they have finished the matching activity, the pack could be shuffled, placed face down, and used for a game of Pelmanism.

For able pupils, add the extension cards to the pack for matching. When they have matched their pairs of cards correctly, they could be given more items of information about the Aliens (or invent their own) and then sketch graphs to show the information, labelling the axes appropriately.

*If time allows, pupils could try the supplementary problem on **Resource 3.5**, an analysis of the distances Aliens have travelled when they become very dangerous*

BOWLAND MATHS

Review

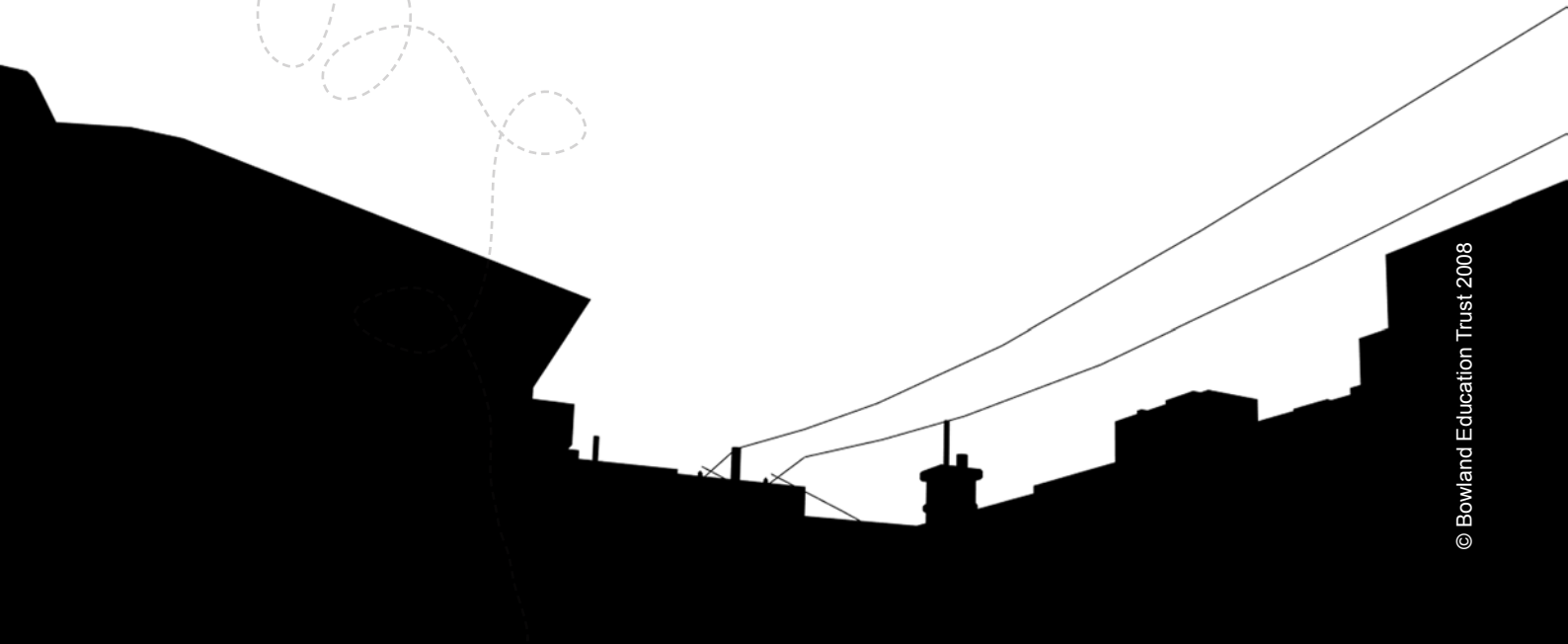
Take feedback on the matching pairs of cards and the labelling of the axes asking pupils to explain their thinking. Draw out that a graph of a particular shape can represent more than one situation.

If you wish, support the discussion with the slides in **Resource 3.6**.

Homework

Play **Resource 3.7**, a 1-minute audio clip. One of the teachers is missing. She went outside to see if she could spot the helicopter and the Aliens have taken her.

Give out copies of **Resource 3.8**, Homework Task 3, choosing Task A or Task B or both, depending on the amount and level of challenge that you want to provide.



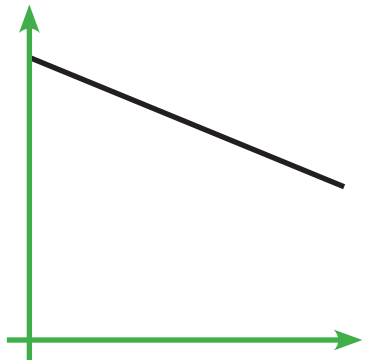
Situation report

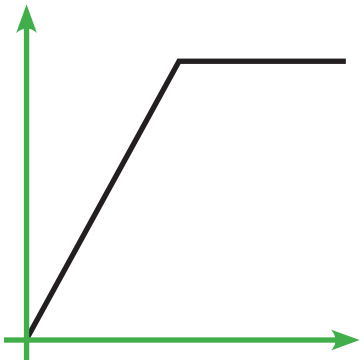
Copy and cut up to make packs of cards

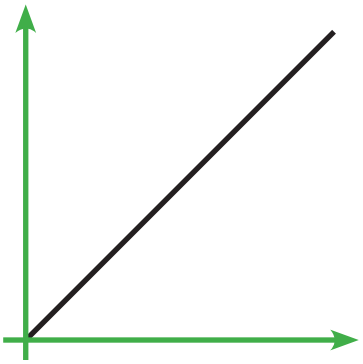
Every hour, each Alien has doubled its mass and then exploded into two.

In the first hour, a green fog spread rapidly around each ship. It stayed there all day but at dusk it started to disappear.

Some Aliens seem to move forward 10 paces then back 3 paces.



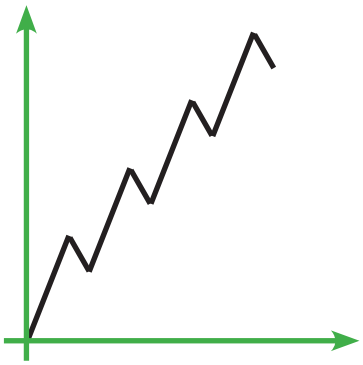


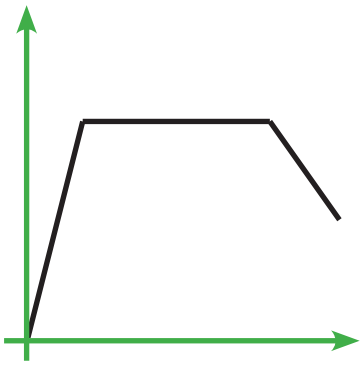


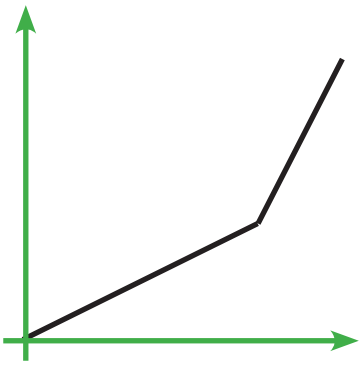
As time has passed, the Aliens' grey skin has steadily become covered in strange brown spots.

The number of Aliens has doubled every hour.

The flag on the Old Town Hall has been lowered to half-mast because so many people have died.



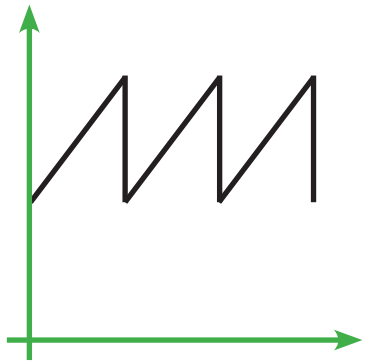




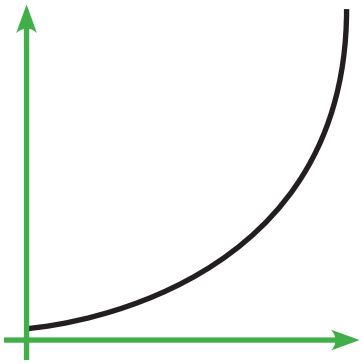
Situation report (continued)

Copy and cut up to make packs of cards

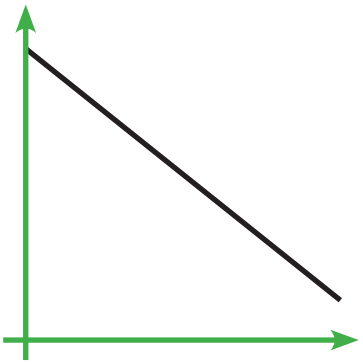
The more people there are in a group, the less interest the Aliens seem to take.



Each tentacle grew steadily until it was 3 metres long.

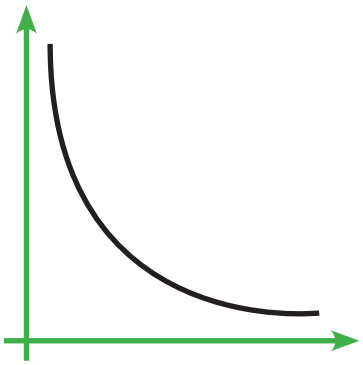


I watched an Alien move at a steady speed on the level ground but it speeded up when it went downhill.

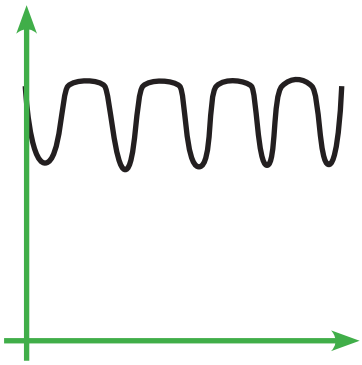


Extension cards

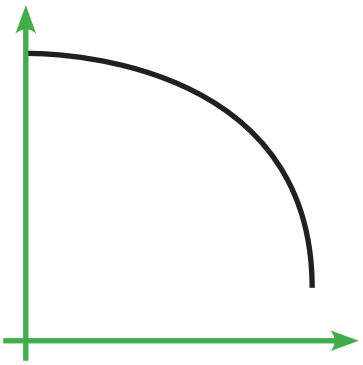
The bus from the Car Park to the Imperial Hotel kept a steady speed but slowed down for corners in case it met Aliens.



The light is fading more quickly now. Soon it will be dark.

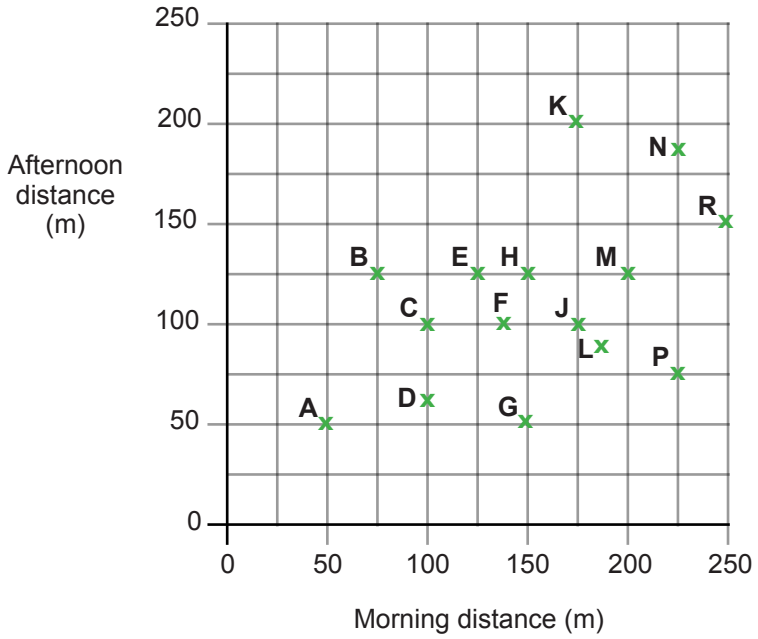


We shared the food equally among us. The more of us that shared the food, the less we had to eat.



Supplementary problem 3

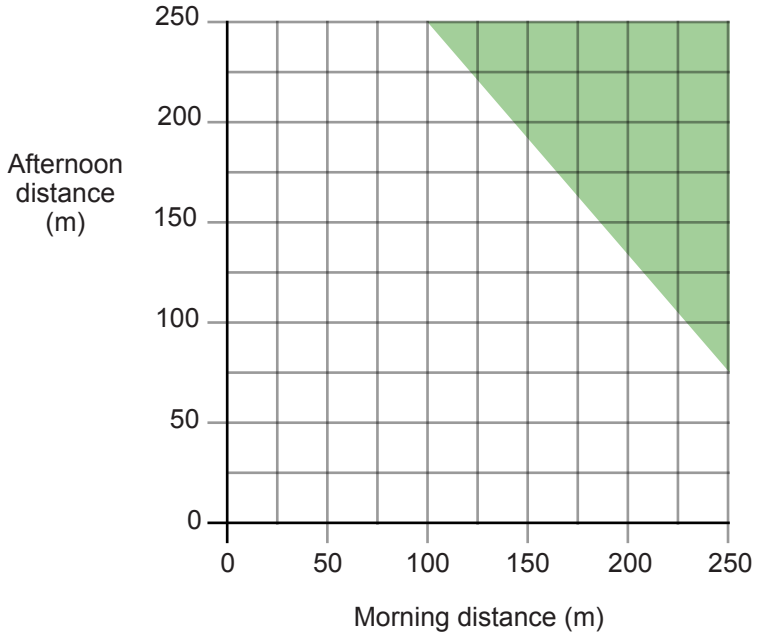
The scatter graph shows how far 15 Aliens have travelled from their space ships.



To find the total distance travelled by an Alien, add the morning distance to the afternoon distance.

1 Which Alien has travelled furthest?

2 One teacher says:
'Aliens in the shaded region on the graph have become very dangerous.'



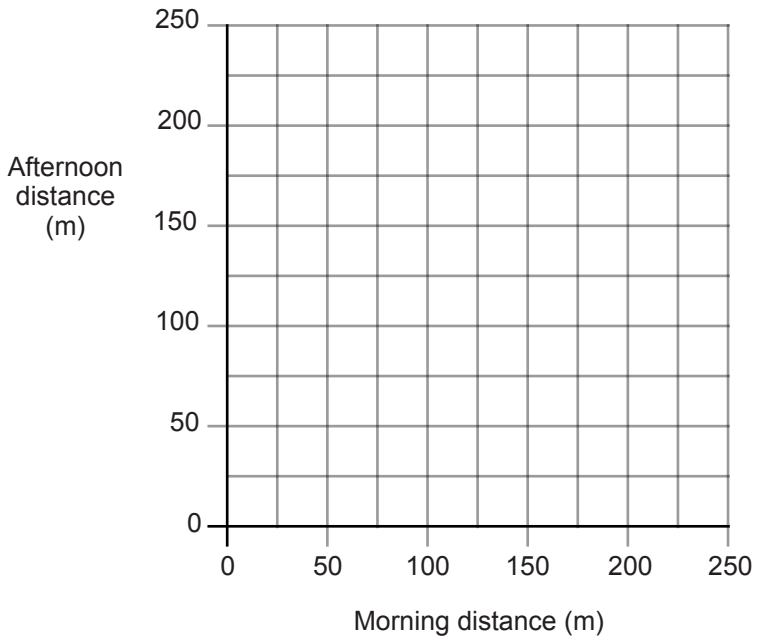
What is the least total distance that an Alien travels to become very dangerous?

Supplementary problem 3 (continued)

3 Another teacher thinks differently. This teacher says:

'Aliens become very dangerous when
the morning distance is 125 metres or more, and
the afternoon distance is 125 or more, and
the total distance is 325 metres or more.'

On the graph below, shade the region that shows when Aliens become very dangerous.



4 A third teacher says:

'In general, the Aliens travelled further in the morning than in the afternoon.
They must be getting tired.'

Do you agree with the teacher?
Explain your answer.

Homework task 3

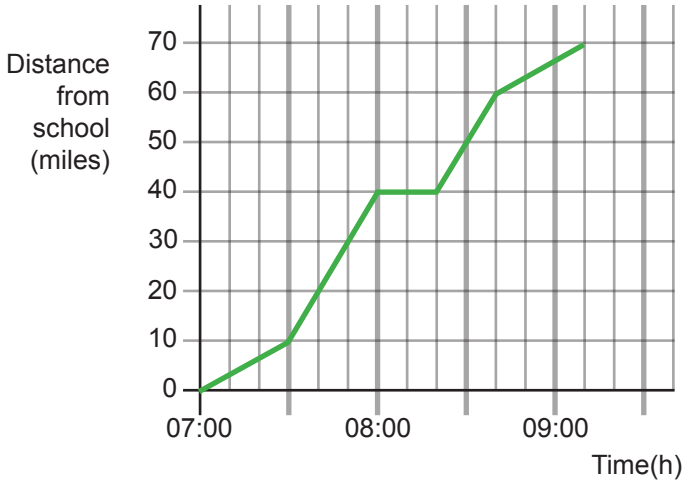
Task A

1 Sketch a graph to represent this story. Remember to label the axes.

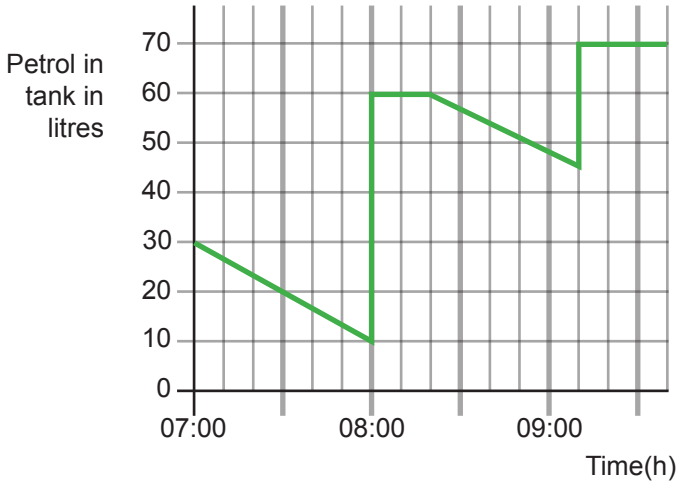
The spaceships came down rapidly through the atmosphere until they were over Manford City.
They hovered overhead for several minutes before descending slowly to the ground.

Task B

2 This graph of the bus journey from school to Manford Car Park shows the distance travelled.



This graph of the same journey shows the amount of fuel in the petrol tank of the bus.



a What time was it when the bus reached the Car Park in Manford?

b Between what times did the bus have less than 20 litres of fuel in its tank?

c The driver bought petrol at the Motorway Service Station and the Car Park Garage. At which of these did he buy more fuel?

d Estimate the total amount of petrol that the bus driver bought.

e How much petrol did the bus use over the entire journey?

BOWLAND MATHS

Narrative

The teacher who has been captured is being held in the mother ship. From her cell, the teacher can see the massive brain that controls all the Aliens and the vast triangular communicator that sends signals to the Aliens in the city. The teacher thinks that the communicator uses a code based on patterns. If she could get out of her cell and crack the code to the communicator, she could send a false communication to the Aliens to entice them back to their ships.

Problems

- What is the code that will unlock the cell?
- What is the code for the patterns used by the communicator?

Mathematics Content Objectives

- Generate integer sequences (levels 4 and 5)
- Find and describe the *n*th term of a sequence (level 6)
- Formulate and solve linear equations (level 6)

Learning points

- Terms in a linear sequence increase or decrease by the same amount each time.
- The formula for the *n*th term is the position-to-term rule for the sequence.
- You can generate the terms of a sequence by substituting values of *n* into the formula for the *n*th term.

Alien invasion resources

4.1 Slides: Sequences for analysis

4.2 Video clip: film of scared captured teacher giving the background to the Alien communicator and explaining what is required in the lesson (3 minutes)

4.3 A4 resource sheets of the problems of the Alien communicator and the switches (print one set per pupil)

4.4 Optional A4 resource sheet with a supplementary problem for groups that finish quickly

4.5 Optional video clip: Breaking news about overseas developments (4 minutes)

For pupils: calculators.

BOWLAND MATHS

Main activity

Either before or at the start of the lesson, practise generating sequences of numbers given a formula for the *n*th term such as $5n - 2$, or 3^n. Also show or remind pupils how to find the nth term of a sequence based on patterns of shapes. Either display the sequences on **Resource 4.1**, **slides 1 and 2**, or use your own materials. Use prompts such as:

- Could you represent this sequence of shapes in a table? What number patterns can you see in the table? Explain why the number patterns arise.
- What do you think the 20th shape in the sequence will be like. How can you test your prediction?
- What formula would describe what you have discovered?

Introduce the last stage of the invasion. From her cell next to the communicator, the captured teacher can reach its keypad. Play **Resource 4.2**, a 3-minute video clip of the teacher's message. Her suggestion is that she should try to send out a message to entice the Aliens back to their ships. She needs to know how the communicator works and send the message.

Give out copies of **Resource 4.3** for groups to work on the problems. In order to gain a sense of achievement, all pupils need to crack the final part of the coded message that will lure away the Aliens. *If time is short, some or all pupils could omit problems e and f on Resource 4.3 and go straight to problem g.*

If time allows, groups could try the supplementary problem on Resource 4.4 to help the teacher to escape. If they get stuck, you could prompt them with:

- *Would it help to list the multiples of the numbers on the keypad?*

They can then look for combinations of multiples that sum to the required total.

Differentiation

Some pupils may be interested in looking further into the patterns in Pascal's triangle, e.g. the patterns of triangular numbers and of Fibonacci numbers (see **Useful websites** in the introductory notes). Pupils could also explore the 'routes' that can be taken on Pascal's triangle and the links with probability.

Review

Take feedback on solutions to the problems and strategies for solving them.

You could explain that the triangle of numbers on the communicator is named in the English-speaking world after the Frenchman Blaise Pascal (1623–1662), although mathematicians in Iran, China, India, and Italy discovered it years before.

Also ask:

- Why do you think that are we working in this way in maths lessons?

Draw out the mathematical theme linking the lessons in this unit: that maths plays a vital part in communications, not only through maps, diagrams and graphs but also through formulae and codes. Explain that the exciting real mathematics involved in many jobs requires working in teams to solve real problems, just as would happen if there really were an alien invasion.

*If time allows, finish by playing **Resource 4.5**, a 4-minute video. Globe news reports that the captured teacher has rejoined the class and everyone has been lifted out by the Army's helicopter. The Aliens seem to have gone but there are new reports of four large shadows over Sydney in Australia and of a strange signal coming from a Russian satellite … A research unit has been set up. Anyone who has experience of dealing successfully with the Aliens should contact the unit on 0801 242424.*

Assessment and homework

You could list the objectives for the unit in pupil-friendly wording so that pupils can make their own assessment of their progress, perhaps using a traffic-light system.

The Key Stage 3 test questions listed below relate to the topics touched on in this unit. You could choose some of the questions to suit the groups of pupils in your class, either to try as homework or in a further lesson.

Level 5
2003 Paper A1 q. 17
2001 Paper A2 q. 12
2000 Paper A2 q. 16
2000 Paper A1 q. 13
1997 Paper A2 q. 12
1995 Paper A1 q. 9

Level 6
2006 Paper D1 q. 2
2006 Paper D2 q. 4
2005 Paper D1 q. 8
2001 Paper B2 q. 12
2000 Paper B2 q. 12
1996 Paper C2 q. 7

Level 7
2006 Paper D2 q. 9
2003 Paper D1 q. 9
1997 Paper B1 q. 10
1997 Paper D2 q. 9

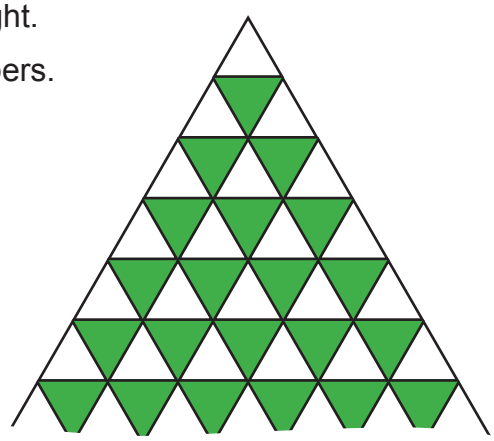
RESOURCE SHEET 4.3
Cracking the codes

Cracking the codes

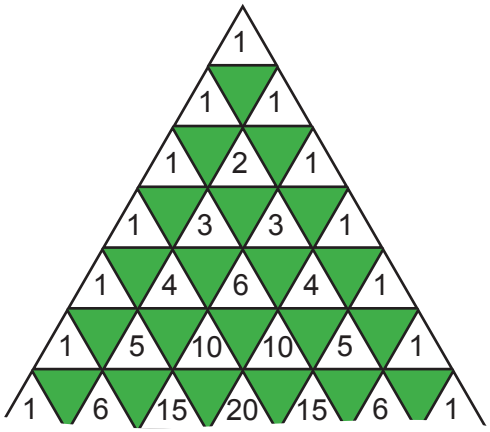
A teacher has been captured by the Aliens and locked in the mother ship!

She is next to the huge communicator with a pattern of small triangles.
She can see only the first 7 rows. The rest of it is out of sight.

The keypad next to the communicator is for entering numbers.



a To switch on the communicator, the teacher has to type in the total number of small triangles in the 100th row.
What number should she type in? _____



b It works! The communicator is switched on!
Immediately, a number starts to flash in each of the small white triangles.
Explain how the number pattern builds up.

c The teacher can see only up to the 7th row.
What are the numbers in the next two rows?

_____

_____

d The teacher knows that to send a message she has to type in the total of the numbers in one of the rows of the communicator. To help her, complete this table.

Row	1	2	3	4	5	6	7
Total of all the numbers in the row	1	2	4				

Use the table to help you to work out a formula for the total of the numbers in the *n*th row of the communicator.

e Messages to the Aliens from the communicator are sent in two parts for security reasons.

To send the first part of her message, the teacher must type in the total of the numbers in the 21st row. Use your calculator to work out this number.

RESOURCE SHEET 4.3
Cracking the codes (continued)

Cracking the codes (continued)

f Each digit of the total stands for a letter.

Use this code-breaker to work out the first part of the message.

0	1	2	3	4	5	6	7	8	9
O	N	P	E	W	A	E	K	M	D

g To work out the second part of the message to the Aliens from the communicator, the teacher must work out the 10th number in this sequence.

11^0 11^1 11^2 11^3 11^4 11^5 ...

By typing in the 10th number in the sequence, the teacher will send the second part of the message to the Aliens.

Think how you could work out the 10th term of this sequence.

Use your calculator to work out this number.

Each digit of the number stands for a letter.
Now use this code-breaker to work out the second part of the message.

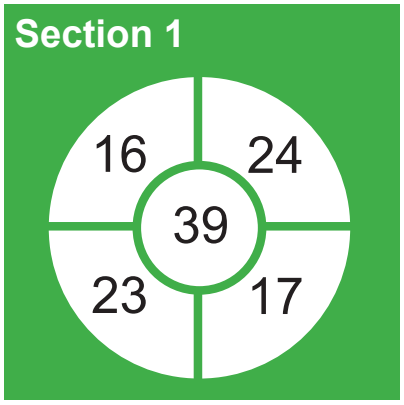
0	1	2	3	4	5	6	7	8	9
C	N	F	A	E	S	U	T	H	R

Supplementary problem 4

There is a massive door to the cell in which the teacher has been held.
It has an electronic combination lock which has **three sections**.

The teacher has watched the Aliens.
She knows that the total of the numbers pressed in any section is always 100.

1 Here is the first section. There are five keys with numbers on them.

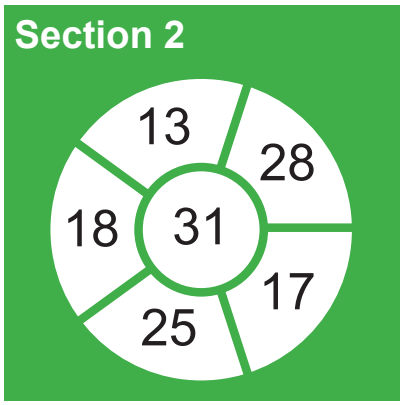


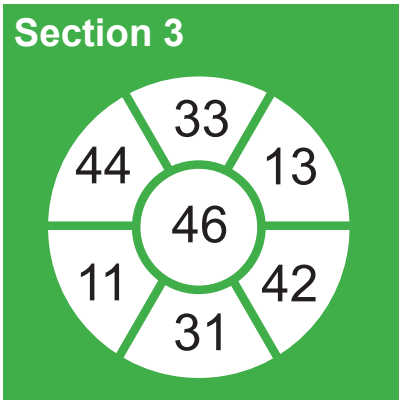
Only two of the keys can be pressed in any section but they can be pressed as many times as you like.

If the wrong key is pressed, the space ship will self-destruct!
If the correct numbers are pressed, the teacher can escape.

Which two keys should the teacher press in this section to make 100?
How many times should she press each of the two keys?

2 Here are sections 2 and 3 of the combination lock.





There are six keys in section 2 and seven keys in section 3.

a Which two keys should the teacher press in section 2 to make 100?
How many times should she press each of the two keys?

b Which two keys should the teacher press in section 3 to make 100?
How many times should she press each of the two keys?